8. Technological shift and labour markets

KAT.TAL.322 Advanced Course in Labour Economics

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September 17, 2025

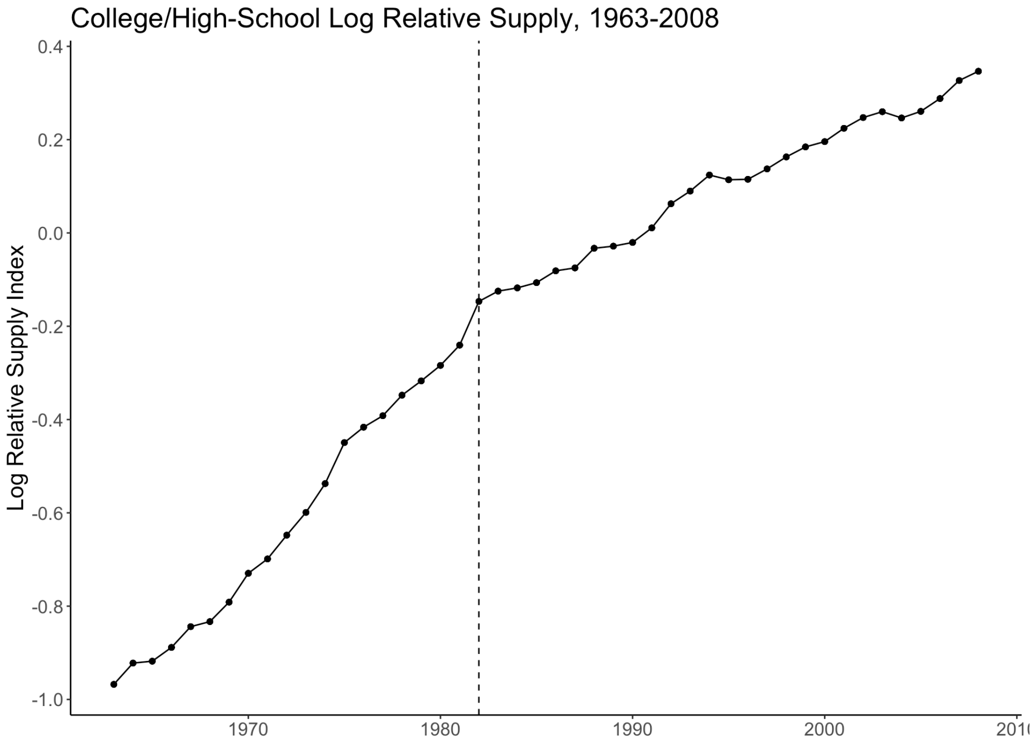
Technological shift and the labour market

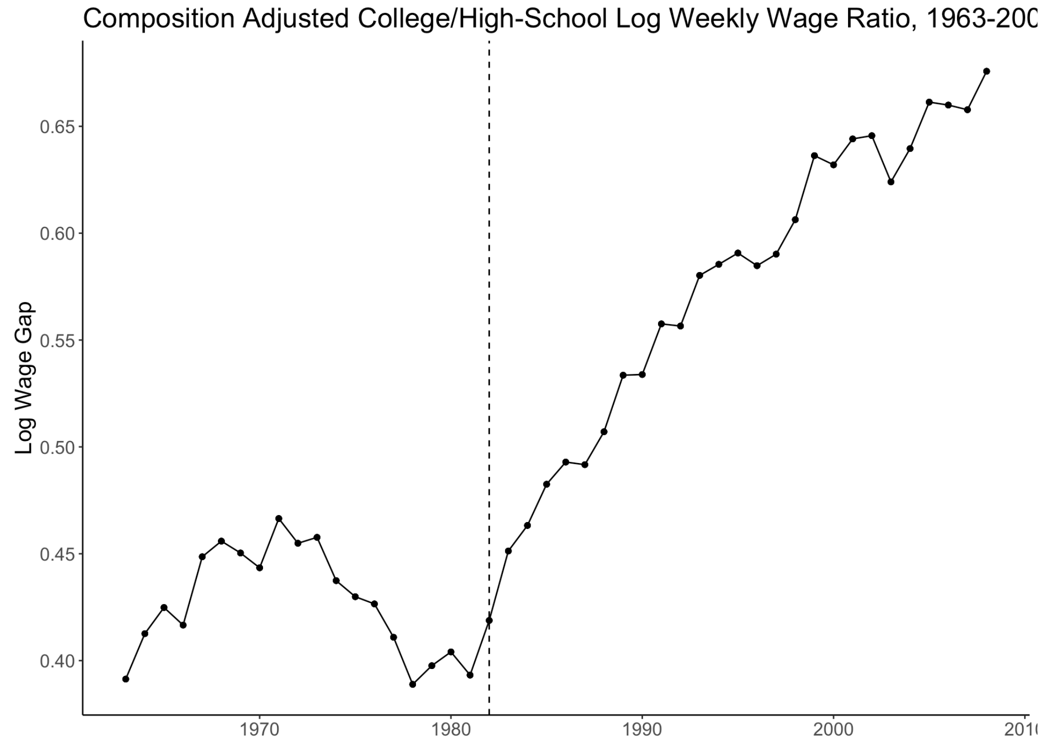
Today

- Stylised facts
- Canonical model
- Task-based model
- Empirical results

Stylised facts

Labour market of educated workers





Source: Figures 1 and 2 (Acemoglu and Autor 2011)

Canonical model

Canonical model

Overview

- Two types of labour: high- and low-skill
  Typically, high edu and low edu (can be relaxed)
- Skill-biased technological change (SBTC)
  New technology disproportionately $\uparrow$ high-skill labour productivity
- High- and low-skill are imperfectly substitutable
  Typically, CES production function with elasticity of substitution σ
- Competitive labour market

Canonical model

Production function

$$Y = \left[(A_L L)^{\frac{\sigma-1}{\sigma}} + (A_H H)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- A_L and A_H are **factor-augmenting** technology terms
- $\sigma \in [0, \infty)$ is the elasticity of substitution
  - → $\sigma > 1$ gross substitutes
  - → $\sigma < 1$ gross complements
  - → $\sigma = 0$ perfect complements (Leontieff production)
  - → $\sigma \longrightarrow \infty$ perfect substitutes
  - → $\sigma = 1$ Cobb-Douglas production

Canonical model

Rationalisation of CES production function

1. Single output Y; H and L are imperfect substitutes
2. Two goods $Y_H = A_H H$ and $Y_L = A_L L$; CES utility of consumers
   $\left[Y_L^{\frac{\sigma-1}{\sigma}} + Y_H^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$
3. Combination of the 1. and 2.

Supply of H and L assumed inelastic $\Rightarrow$ study only firm side

Canonical model

Equilibrium wages

$$w_L = A_L^{\frac{\sigma-1}{\sigma}} \left[A_L^{\frac{\sigma-1}{\sigma}} + A_H^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}$$

$$w_H = A_H^{\frac{\sigma-1}{\sigma}} \left[A_L^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L} \right)^{-\frac{\sigma-1}{\sigma}} + A_H^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}$$

Comparative statics:

- $\frac{\partial w_L}{\partial H/L} > 0$ low-skill wage rises with $\frac{H}{L}$
- $\frac{\partial w_H}{\partial H/L} < 0$ high-skill wage falls with $\frac{H}{L}$
- $\frac{\partial w_i}{\partial A_L} > 0$ and $\frac{\partial w_i}{\partial A_H} > 0, \ \forall i \in \{L, H\}$

Canonical model

Skill premium

$$\frac{w_H}{w_L} = \left(\frac{A_H}{A_L}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}$$

Δ relative supply

$$\frac{\partial \ln \frac{w_H}{w_L}}{\partial \ln \frac{H}{L}} = -\frac{1}{\sigma} < 0$$

Δ technology

$$\frac{\partial \ln \frac{w_H}{w_L}}{\partial \ln \frac{A_H}{A_L}} = \frac{\sigma - 1}{\sigma} \lesseqgtr 0$$

- Gross substitutes: $\sigma > 1 \Rightarrow \frac{\partial \ln w_H/w_L}{\partial \ln A_H/A_L} > 0$
- Gross complements: $\sigma < 1 \Rightarrow \frac{\partial \ln w_H/w_L}{\partial \ln A_H/A_L} < 0$
- Cobb-Douglas: $\sigma = 1 \Rightarrow \frac{\partial \ln w_H/w_L}{\partial \ln A_H/A_L} = 0$

Tinbergen's race in the data

*Katz and Murphy (**1992**)*

The log-equation of skill premium is extremely attractive for empirical analysis

$$\ln \frac{w_{H,t}}{w_{L,t}} = \frac{\sigma - 1}{\sigma} \ln\left(\frac{A_{H,t}}{A_{L,t}}\right) - \frac{1}{\sigma} \ln\left(\frac{H_t}{L_t}\right)$$

Assume a log-linear trend in relative productivities

$$\ln\left(\frac{A_{H,t}}{A_{L,t}}\right) = \alpha_0 + \alpha_1 t$$

and plug it into the log skill premium equation:

$$\ln \frac{w_{H,t}}{w_{L,t}} = \frac{\sigma - 1}{\sigma}\alpha_0 + \frac{\sigma - 1}{\sigma}\alpha_1 t - \frac{1}{\sigma} \ln\left(\frac{H_t}{L_t}\right)$$

Tinbergen's race in the data

*Katz and Murphy (**1992**)*

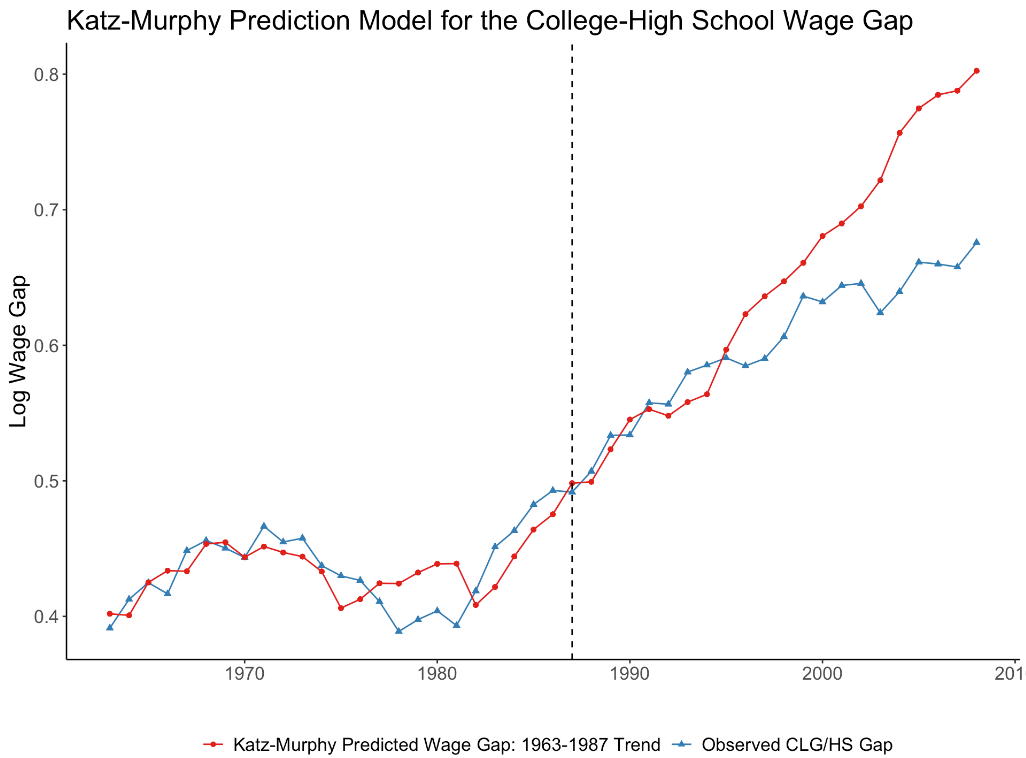
Estimated the skill premium equation using the US data in 1963-87

$$\ln \omega_t = \text{cons} + \underset{(0.005)}{0.027} \times t - \underset{(0.128)}{0.612} \times \ln\left(\frac{H_t}{L_t}\right)$$

Implies elasticity of substitution $\sigma \approx \frac{1}{0.612} = 1.63$

Agrees with other estimates that place σ between 1.4 and 2 (Acemoglu and Autor 2011)

Tinbergen's race in the data



Source: Figure 19 (Acemoglu and Autor 2011)

Very close fit up to mid-1990s, diverge later

Fit up to 2008 implies $\sigma \approx 2.95$

Accounting for divergence:

- non-linear time trend in $\ln \frac{A_H}{A_L}$ brings σ back to 1.8, but implies $\frac{A_H}{A_L}$ slowed down
- differentiate labour by age/experience as well

Canonical model

Summary

1. Simple link between wage structure and technological change
2. Attractive explanation for college/no college wage inequality[1]
3. Average wages ↑ (follows from $\partial w_i / \partial A_H$ and $\partial w_i / \partial A_L$)

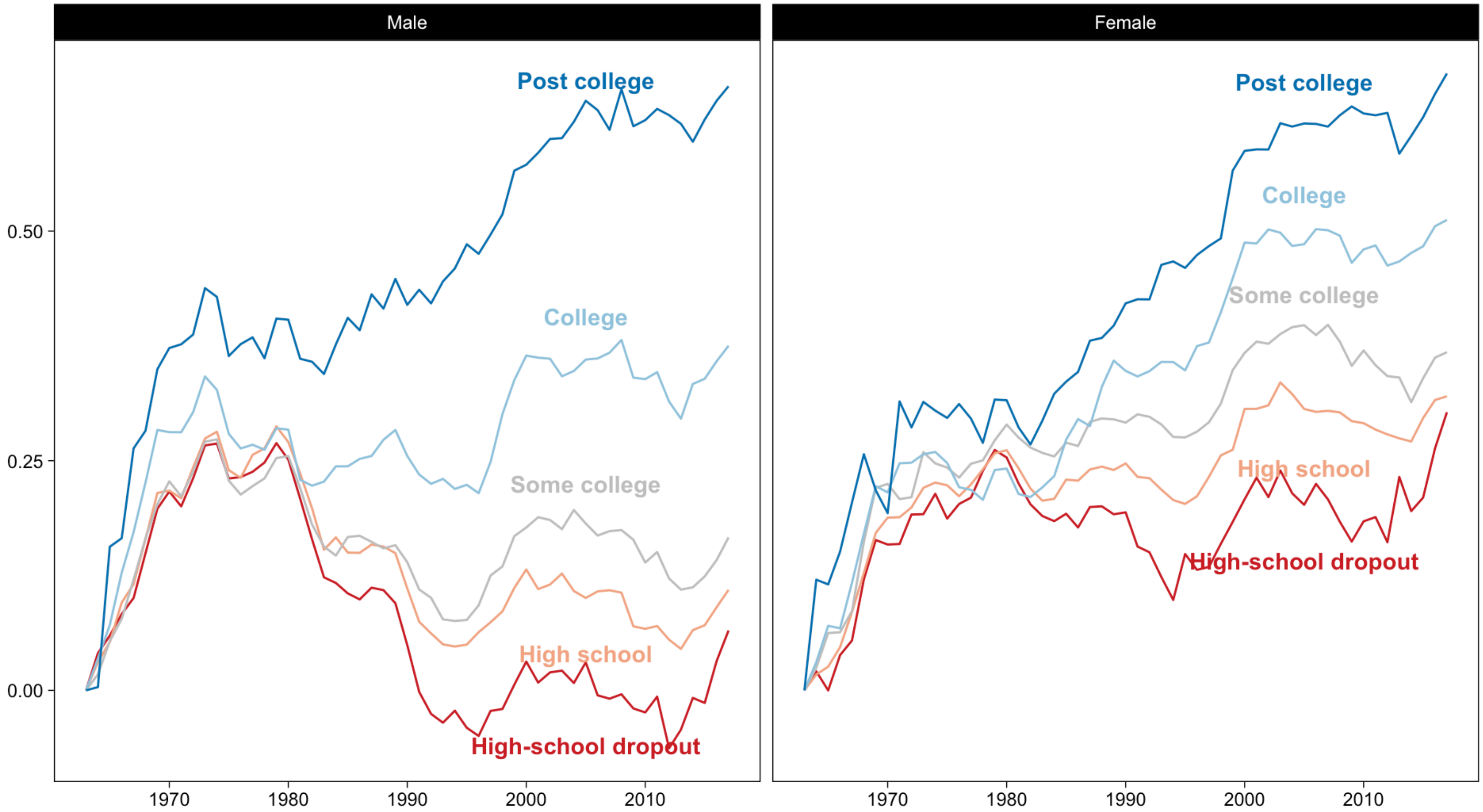
However, the model cannot explain other trends observed in the data:

1. Falling w_L
2. Earnings polarization
3. Job polarization

Also silent about endogeneous adoption or labour-replacing technology.

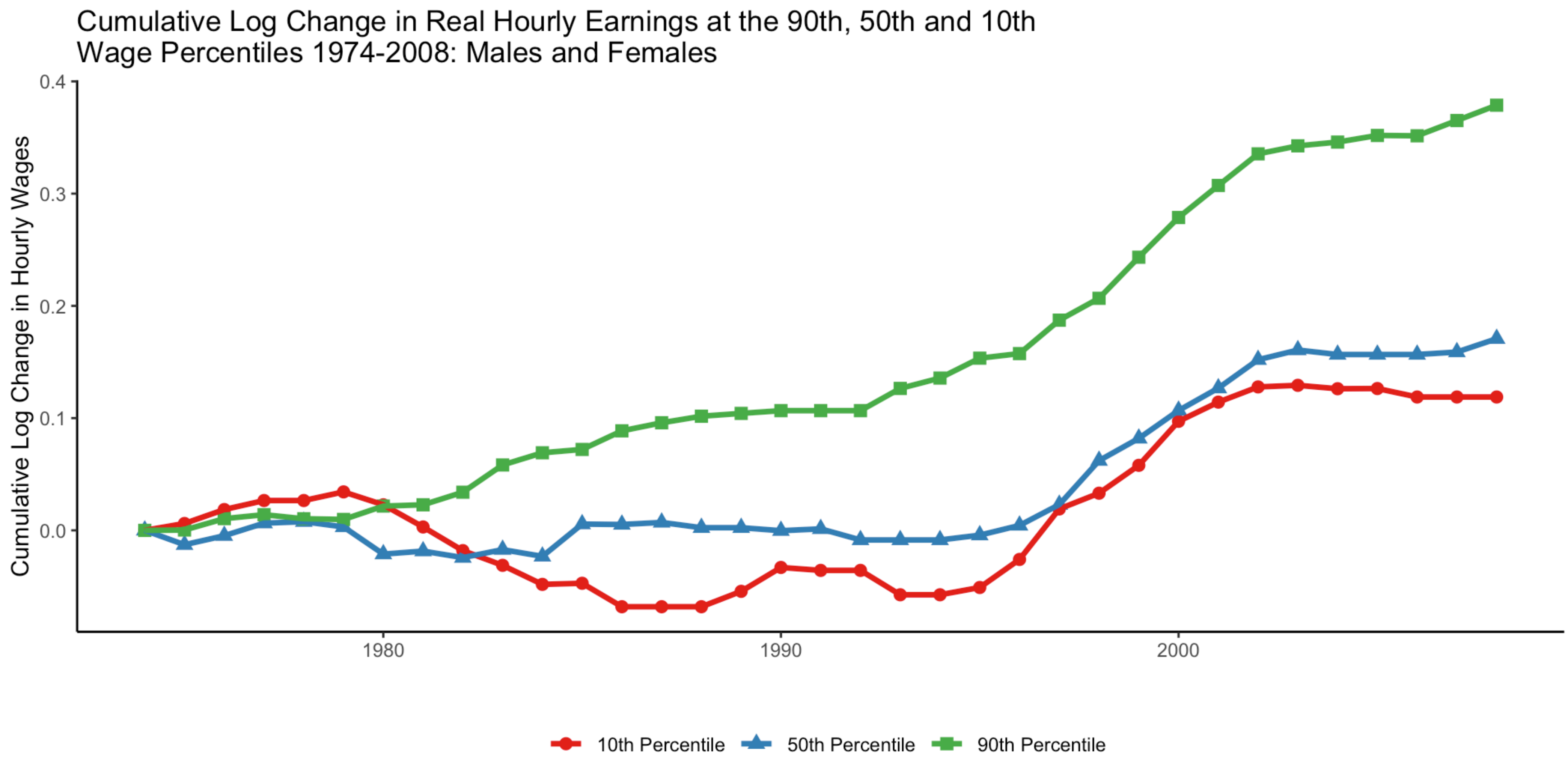
1 For the discussion of this point, see section 3.4 in Acemoglu and Autor (2011)

Unexplained trend: falling real wages



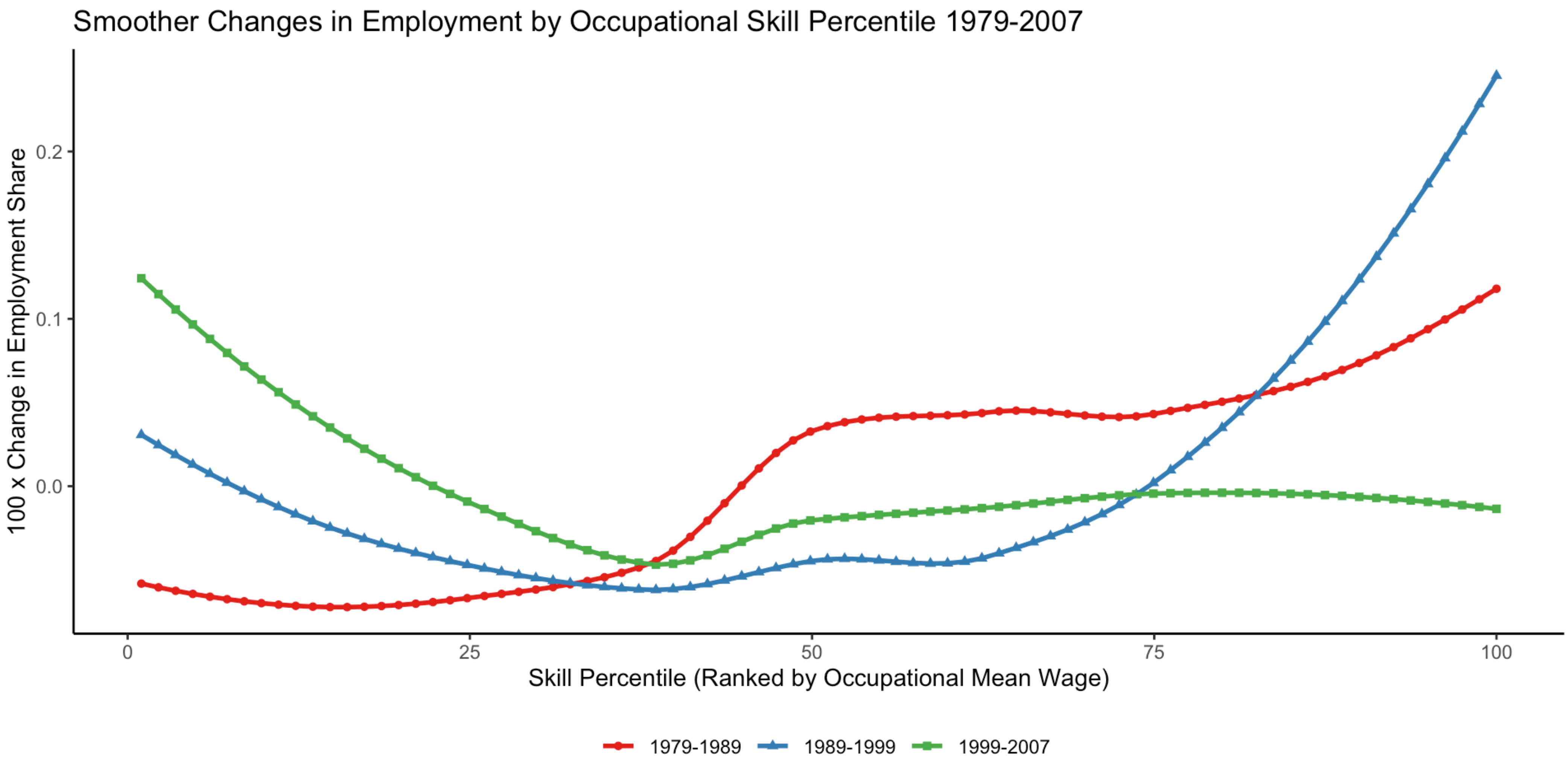
Source: Figure 1 (Autor 2019)

Unexplained trend: earnings polarization



Source: Figure 8 (Acemoglu and Autor 2011)

Unexplained trend: job polarization



Source: Figure 10 (Acemoglu and Autor 2011)

Task-based model

Task-based model

Overview

Task is a unit of work activity that produces output

Skill is a worker's endowment of capabilities for performing tasks

Key features:

1. Tasks can be performed by various inputs (skills, machines)
2. Comparative advantage over tasks among workers
3. Multiple skill groups
4. Consistent with canonical model predictions

Task-based model

Production function

Unique final good Y produced by continuum of tasks $i \in [0,1]$

$$Y = \exp\left[\int_0^1 \ln y(i) \mathrm{d}i\right]$$

Three types of labour: H, M and L supplied inelastically.

$$y(i) = A_L \alpha_L(i) l(i) + A_M \alpha_M(i) m(i) + A_H \alpha_H(i) h(i) + A_K \alpha_K(i) k(i)$$

- A_L, A_M, A_H, A_K are factor-augmenting technologies
- $\alpha_L(i), \alpha_M(i), \alpha_H(i), \alpha_K(i)$ are task productivity schedules
- $l(i), m(i), h(i), k(i)$ are production inputs allocated to task i

Fir simplicity, assume $\alpha_K(i) = 0, \forall i \in [0,1]$. See Acemoglu and Restrepo (2018) for a more complete analysis of the model.

Task-based model

Comparative advantage assumption

$\alpha_L(i)/\alpha_M(i)$ and $\alpha_M(i)/\alpha_H(i)$ are continuously differentiable and strictly decreasing.

Market clearing conditions

$$\int_0^1 l(i)\mathrm{d}i \leq L \qquad \int_0^1 m(i)\mathrm{d}i \leq M \qquad \int_0^1 h(i)\mathrm{d}i \leq H$$

Task-based model

Equilibrium without machines

Lemma 1

Given comparative advantage assumption, there exist I_L and I_H such that

Note that boundaries I_L and I_H are endogenous

This gives rise to the **substitution of skills across tasks**

Task-based model

Law of one wage

Output price is normalised to 1 $\Rightarrow \exp\left[\int_0^1 \ln p(i)\mathrm{d}i\right] = 1$

All tasks employing a given skill pay corresponding wage

$$
\begin{aligned}
w_L &= p(i)A_L\alpha_L(i), & \forall i \in [0, I_L] \\
w_M &= p(i)A_M\alpha_M(i), & \forall i \in (I_L, I_H] \\
w_H &= p(i)A_H\alpha_H(i), & \forall i \in (I_H, 1]
\end{aligned}
$$

See Appendix: derivation of wage equations

Task-based model

Skill allocations

Given the law of one wage, we can show that

$$l(i) = l\left(i'\right) \quad \Rightarrow \quad l(i) = \frac{L}{I_L} \forall i \in [0, I_L]$$

$$m(i) = m\left(i'\right) \quad \Rightarrow \quad m(i) = \frac{M}{I_H - I_L} \forall i \in (I_L, I_H]$$

$$h(i) = h\left(i'\right) \quad \Rightarrow \quad h(i) = \frac{H}{1 - I_H} \forall i \in (I_H, 1]$$

See Appendix: derivation of skill allocations

Task-based model

Endogenous thresholds: no arbitrage

Threshold task I_H: equally profitable to produce with either H or M skills

$$\frac{A_M \alpha_M (I_H) M}{I_H - I_L} = \frac{A_H \alpha_H (I_H) H}{1 - I_H}$$

Similarly, for I_L:

$$\frac{A_L \alpha_L (I_L) L}{I_L} = \frac{A_M \alpha_M (I_L) M}{I_H - I_L}$$

Task-based model

Endogenous thresholds: no arbitrage

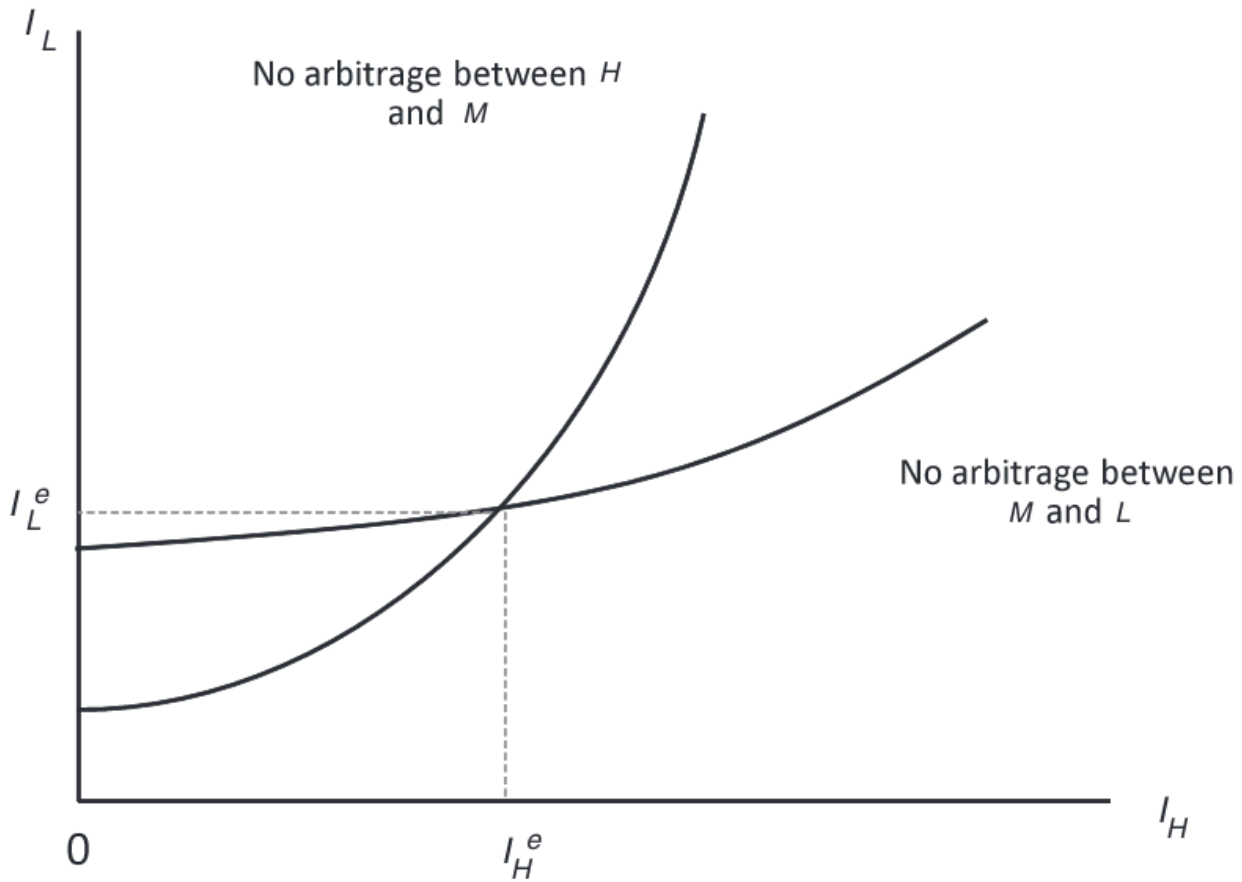


Figure 22 ***Determination of equilibrium threshold tasks.***

Task-based model

Comparative statics: wage elasticities

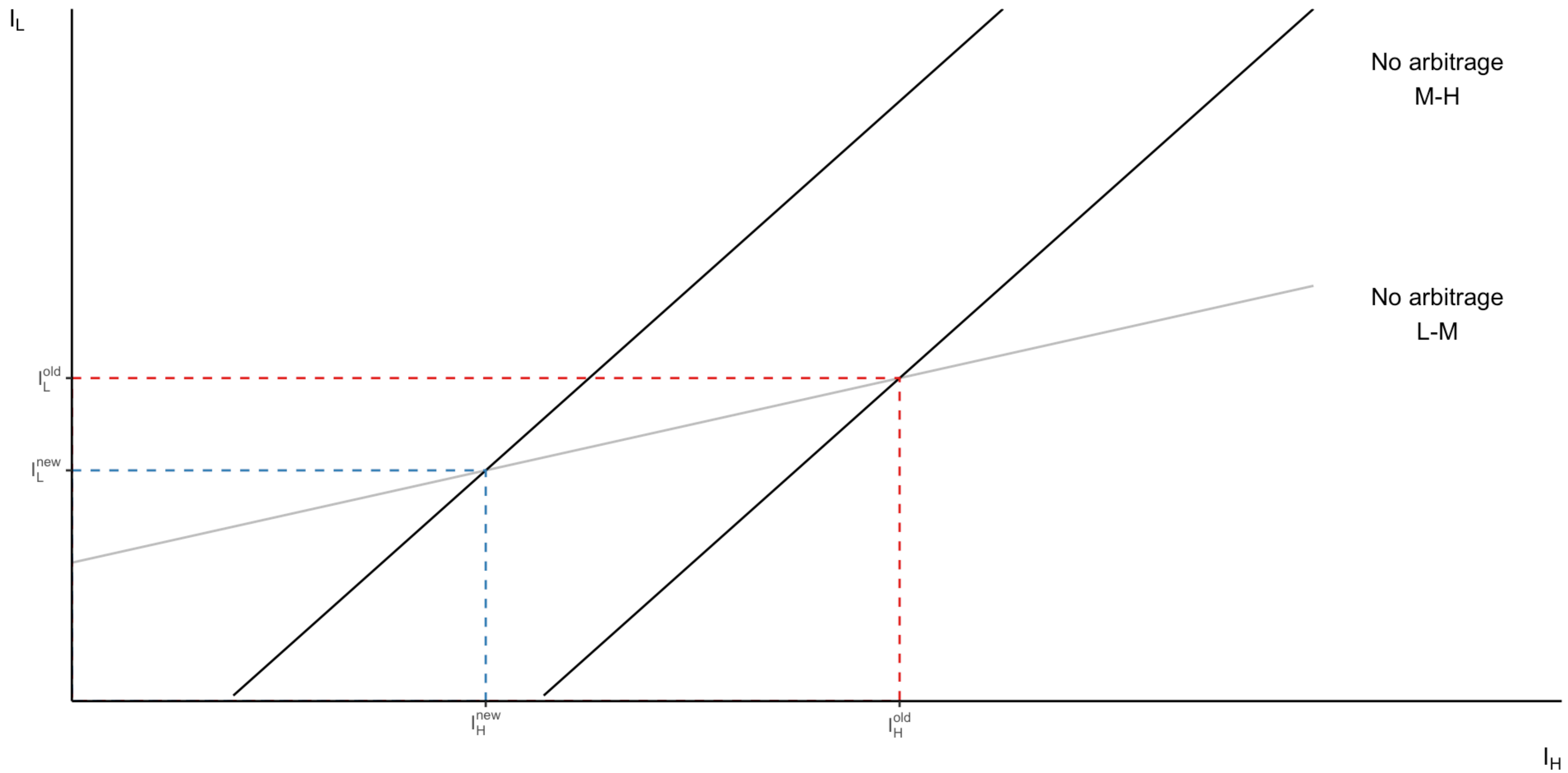
$$\frac{\mathrm{d}\ln w_H/w_L}{\mathrm{d}\ln A_H} > 0 \qquad \frac{\mathrm{d}\ln w_M/w_L}{\mathrm{d}\ln A_H} < 0 \qquad \frac{\mathrm{d}\ln w_H/w_M}{\mathrm{d}\ln A_H} > 0$$

$$\frac{\mathrm{d}\ln w_H/w_L}{\mathrm{d}\ln A_M} \lesseqgtr 0 \qquad \frac{\mathrm{d}\ln w_M/w_L}{\mathrm{d}\ln A_M} > 0 \qquad \frac{\mathrm{d}\ln w_H/w_M}{\mathrm{d}\ln A_M} < 0$$

$$\frac{\mathrm{d}\ln w_H/w_L}{\mathrm{d}\ln A_L} < 0 \qquad \frac{\mathrm{d}\ln w_M/w_L}{\mathrm{d}\ln A_L} < 0 \qquad \frac{\mathrm{d}\ln w_H/w_M}{\mathrm{d}\ln A_L} > 0$$

Task-based model

Comparative statics: $\uparrow A_H$



Source: Figure 25 (Acemoglu and Autor 2011)

Task-based model

Task replacing technologies

Start from initial equilibrium without machines

Assume in $[\underline{I}, \bar{I}] \subset [I_L, I_H]$ machines outperform M. Otherwise, $\alpha_K(i) = 0$.

How does it change the equilibrium?

Task-based model

Task replacing technologies

Assume comparative advantage of H over M stronger than M over L

1. w_H/w_M increases
2. w_M/w_L decreases
3. $w_H/w_L \uparrow (\downarrow)$ if $|\beta'_L(I_L)I_L| \lessgtr |\beta'_H(I_H)(1-I_H)|$

Task-based model

Endogenous supply of skills

Each worker j is endowed with some amount of each skill l^j, m^j, h^j

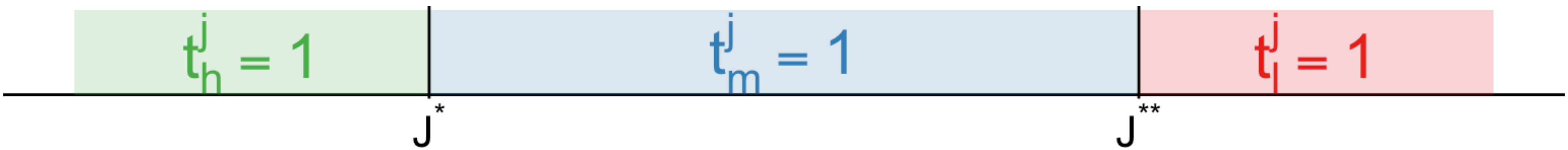
Workers allocate time to each skill given

$$t_l^j + t_m^j + t_h^j \leq 1$$

$$w_L\, t_l^j l^j + w_M\, t_m^j m^j + w_H\, t_h^j h^j$$

Comparative advantage: $\frac{h^j}{m^j}$ and $\frac{m^j}{l^j}$ are decreasing in j

Then, there exist $J^{*}\left(\frac{w_H}{w_M}\right)$ and $J^{**}\left(\frac{w_M}{w_L}\right)$



Task-based model

Illustration in the data

Suppose $\uparrow A_H \Rightarrow \uparrow \frac{w_H}{w_M}, \downarrow \frac{w_M}{w_L}$.

Use occupational specialization at some $t = 0$ as comparative advantage.

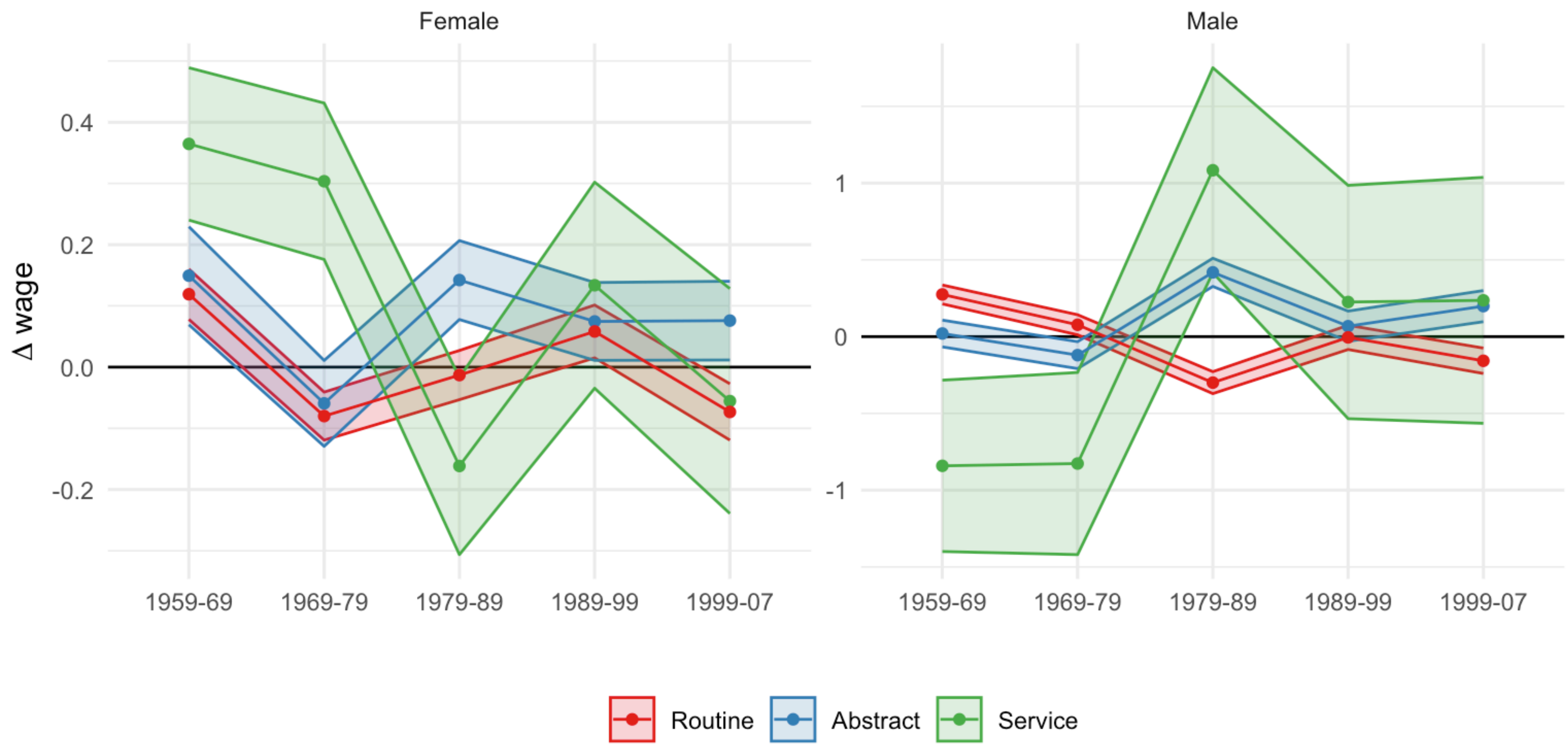
- γ^i_{sejk} share of 1959 population employed in i occupations, $\forall i \in \{H, M, L\}$

$$\Delta w_{sejk\tau} = \sum_t \left[\beta^H_t \gamma^H_{sejk} + \beta^L_t \gamma^L_{sejk}\right] 1\{\tau = t\} + \delta_\tau + \phi_e + \lambda_j + \pi_k + e_{sejk}$$

Descriptive regression informed by the model!

Task-based model

Illustration in the data



Source: Table 10 (Acemoglu and Autor 2011)

Task-based model

Summary

1. A rich model that can accommodate numerous scenarios
    a. Outsourcing tasks to lower-cost countries
    b. Endogenous technological change
    c. Creation of new tasks
2. Useful tool to study effect on inequality and job polarization

Empirical results

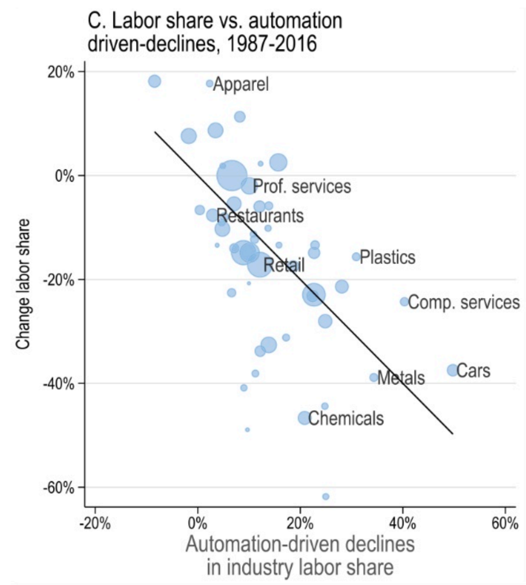
Acemoglu and Restrepo (2022)

Environment

Multi-sector model with imperfect substitution between inputs

$$\text{Task displacement}_g^{\text{direct}} = \sum_{i \in I} \omega_g^i \frac{\omega_{gi}^R}{\omega_i^R} \left(-d\ln s_i^{L,\text{auto}} \right)$$

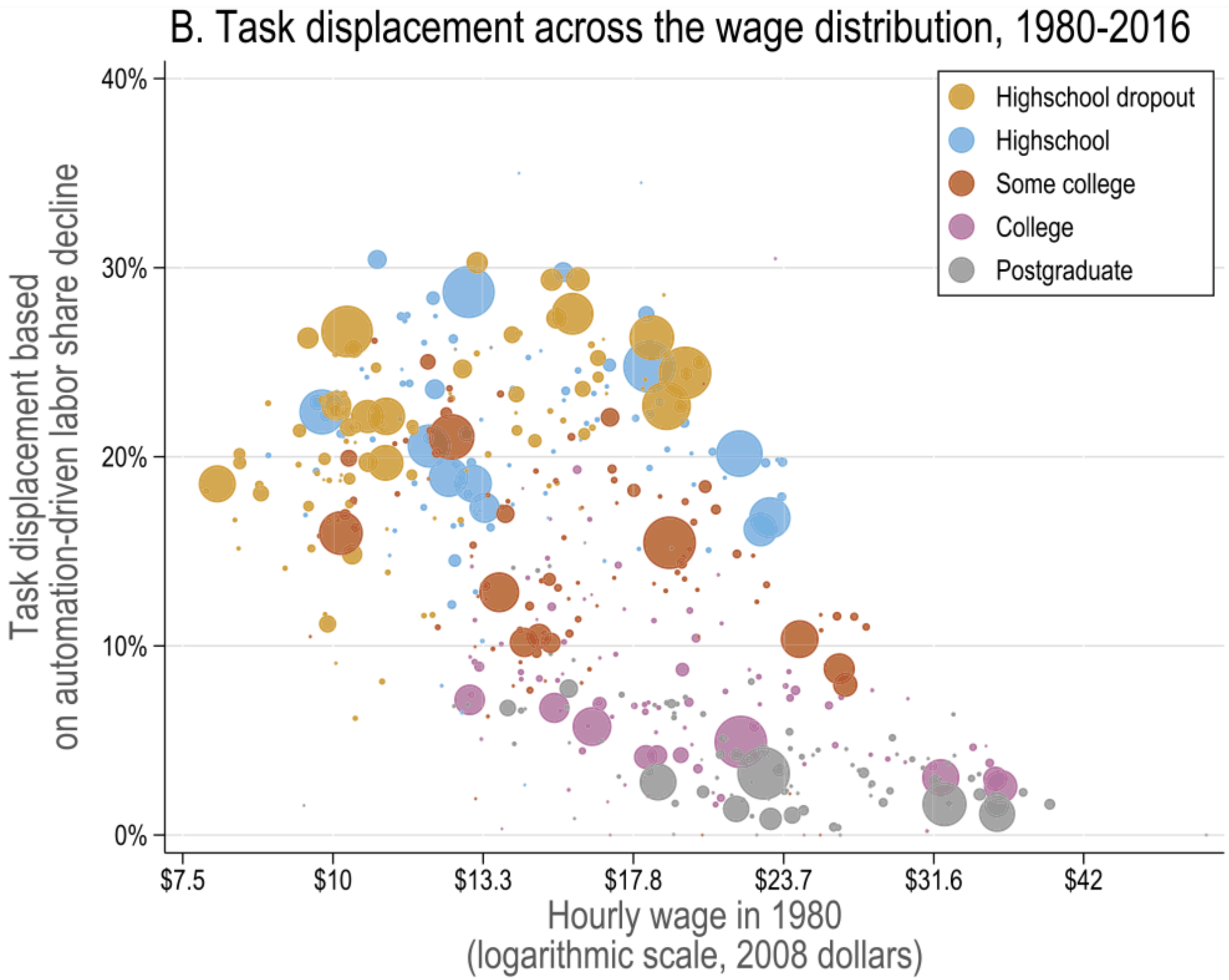
- ω_g^i - share of wages earned by worker group g in industry i (exposure to industry i)at $t = 0$
- $\frac{\omega_{gi}^R}{\omega_i^R}$ - specialization of group g in routine tasks R within industry i at $t = 0$
- $-d\ln s_i^{L,\text{auto}}$ - % decline in industry i's labour share due to automation
    1. attribute 100% of the decline to automation
    2. predict given industry adoption of automation technology



Source: Figure 4 (Acemoglu and Restrepo 2022)

Acemoglu and Restrepo (2022)

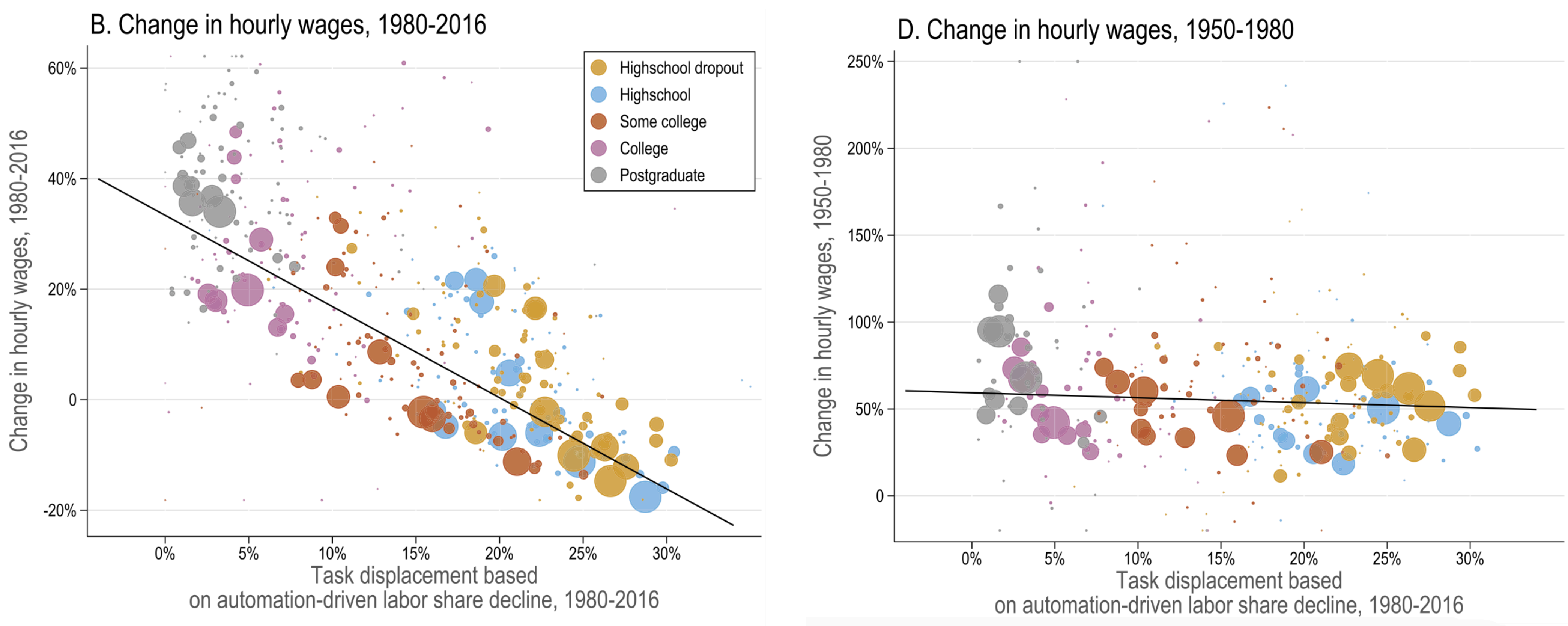
Task displacement



Source: Figure 5

Acemoglu and Restrepo (2022)

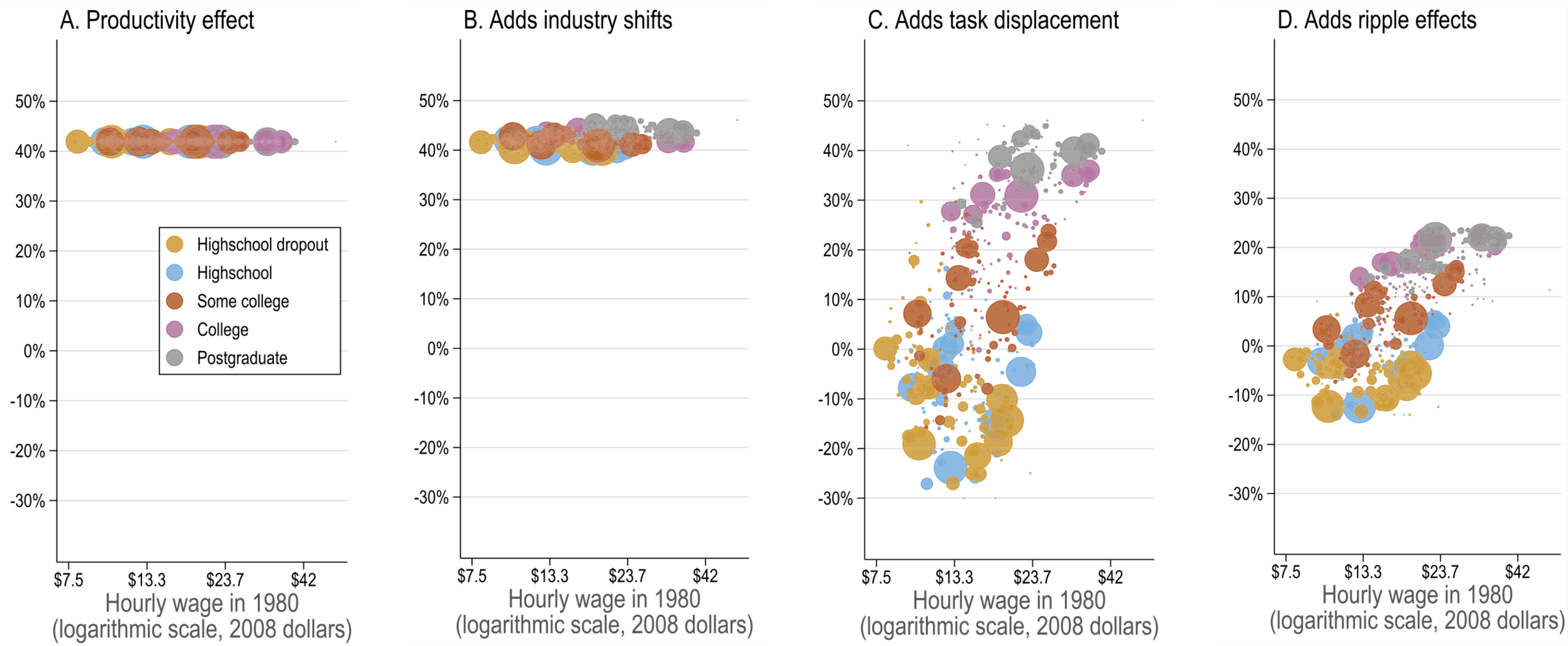
Task displacement and changes in real wages



Source: Figure 6 (Acemoglu and Restrepo 2022)

Acemoglu and Restrepo (2022)

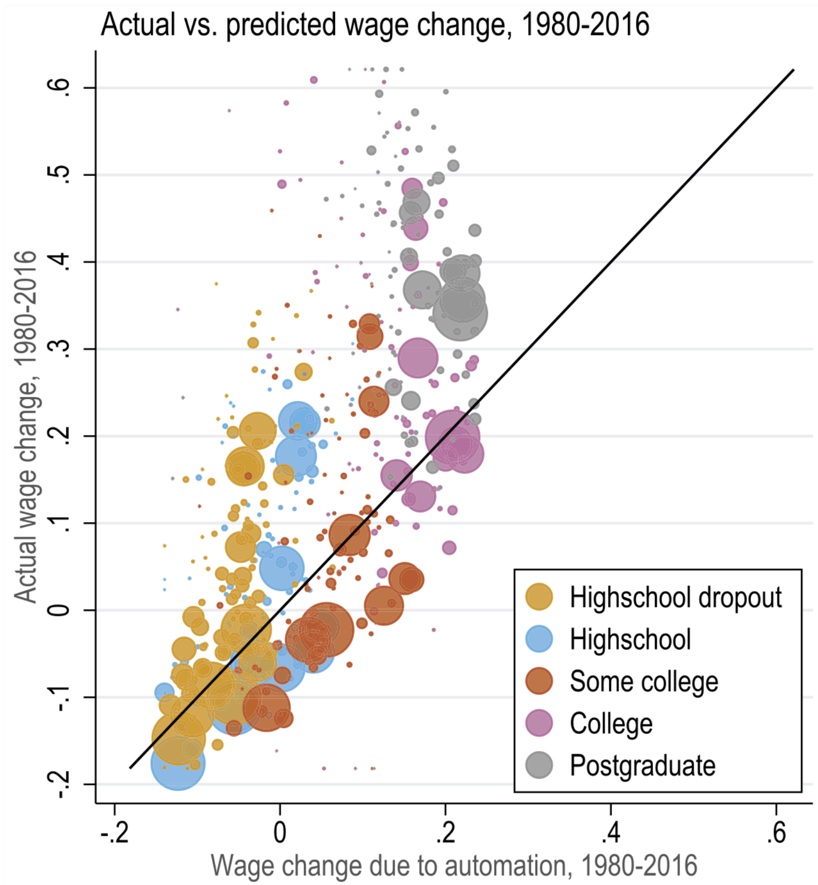
General equilibrium results



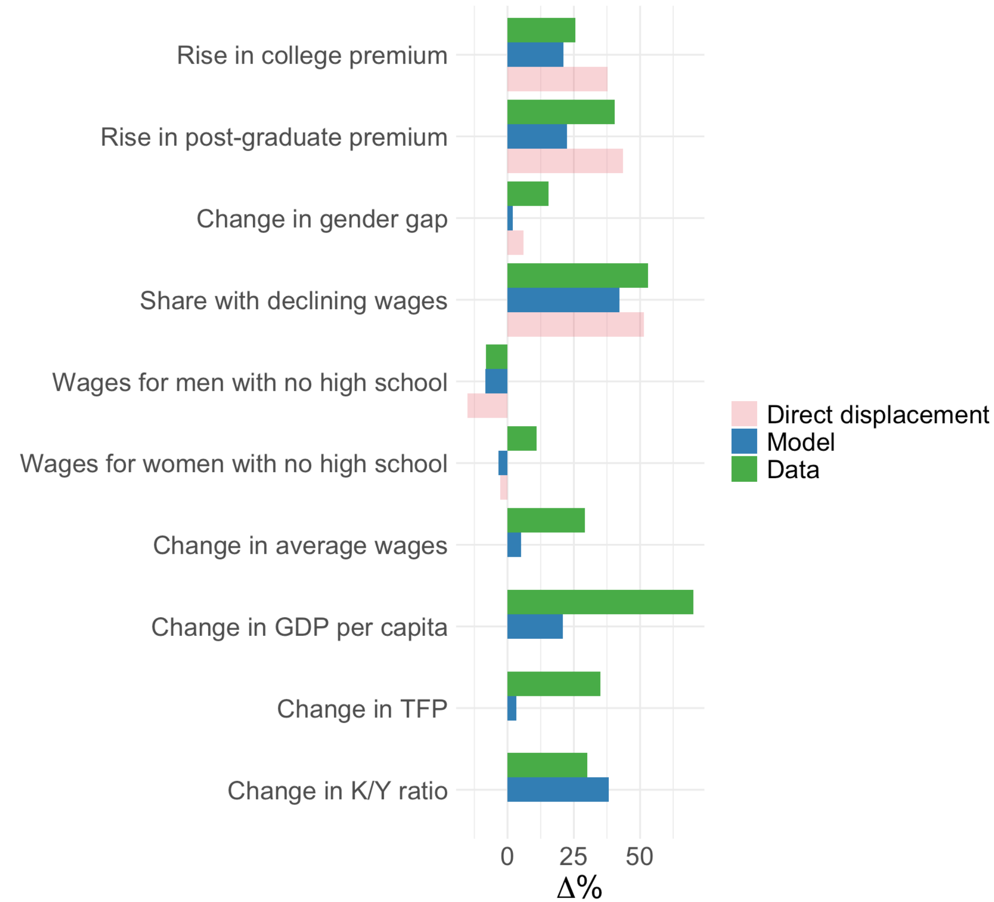
Source: Figure 7 (Acemoglu and Restrepo 2022)

Acemoglu and Restrepo (2022)

Model fit



Source: Figure 8



Source: Table VIII

Summary

Two theories linking technological advancements and labour markets

- Canonical model (SBTC)
  - → Simple application of two-factor labour demand theory
  - → Empirically attractive characterization of between-group inequality
  - → Fails to account for within-group inequality, polarization, and displacement
- Task-based model (automation)
  - → Rich model linking skills to tasks to output
  - → Explains large share of changes in the wage structure since 1980s

Next lecture: Labour market discrimination on 22 Sep

Appendix: derivation of wage equations

The firm problem is to choose entire schedules $(l(i), m(i), h(i))_{i=0}^{1}$ to

$$\max_{(l(i),m(i),h(i))_{i=0}^{1}} PY - w_L L - w_M M - w_H H$$

We normalised $P = 1$. Consider FOC wrt $l(i)$:

$$\frac{Y}{y(i)} A_L \alpha_L(i) = w_L, \qquad \forall i \in [0, I_L]$$

In equilibrium, all L-type workers must be paid same amount ⇒

$$p(i) A_L \alpha_L(i) = w_L, \qquad \forall i \in [0, I_L]$$

Similar argument for w_M and w_H.

Appendix: derivation of skill allocations

Given the law of one price (wage) we can also write that

$$p(i)\alpha_L(i)l(i) = p\left(i'\right)\alpha_L\left(i'\right)l\left(i'\right), \qquad \forall i, i' \in [0, I_L]$$

Given the Appendix: derivation of wage equations, it implies that

$$l(i) = l\left(i'\right) = l, \qquad \forall i, i' \in [0, I_L]$$

Plug it into the market clearing condition for L

$$L = \int_0^{I_L} l(i)\mathrm{d}i = l \cdot I_L \quad \Longrightarrow \quad l(i) = l = \frac{L}{I_L}, \forall i \in [0, I_L]$$

Similar argument for $m(i) = \frac{M}{I_H - I_L}$ and $h(i) = \frac{H}{1 - I_H}$.

References

Acemoglu, Daron, and David Autor. 2011. "Chapter 12 - Skills, Tasks and Technologies: Implications for Employment and Earnings." In *Handbook of Labor Economics*, edited by David Card and Orley Ashenfelter, 4:1043–1171. Elsevier. https://doi.org/10.1016/S0169-7218(11)02410-5.

Acemoglu, Daron, and Pascual Restrepo. 2018. "The Race Between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment." *American Economic Review* 108 (6): 1488–1542. https://doi.org/10.1257/aer.20160696.

———. 2022. "Tasks, Automation, and the Rise in U.S. Wage Inequality." *Econometrica* 90 (5): 1973–2016. https://doi.org/10.3982/ECTA19815.

Autor, David H. 2019. "Work of the Past, Work of the Future." *AEA Papers and Proceedings* 109 (May): 1–32. https://doi.org/10.1257/pandp.20191110.

Autor, David H., Frank Levy, and Richard J. Murnane. 2003. "The Skill Content of Recent Technological Change: An Empirical Exploration*." *The Quarterly Journal of Economics* 118 (4): 1279–1333. https://doi.org/10.1162/003355303322552801.

Katz, Lawrence F., and Kevin M. Murphy. 1992. "Changes in Relative Wages, 1963-1987: Supply and Demand Factors." *The Quarterly Journal of Economics* 107 (1): 35–78. https://doi.org/10.2307/2118323.