3. Labour Demand

KAT.TAL.322 Advanced Course in Labour Economics

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Labour demand

Firm decisions about how much labour to hire

Today

- Static model
- Dynamic model
- Minimum wages and employment

Static model

Static model

Single factor input

Production function $Y = F(L)$ where $F' > 0$ and $F'' < 0$

$$\max_L PF(L) - WL$$

FOC: $F'(L) = \frac{W}{P}$

Downward-sloping labour demand

$$\frac{\partial L}{\partial W} = \frac{1}{PF''(L)} < 0$$

Static model

Two factor inputs: conditional factor demand

Production function $Y = F(L, K)$ where $F_L > 0, F_K > 0, F_{LL} < 0, F_{KK} < 0$

Cost minimization problem: $\min_{L,K} C(L, K) = WL + RK$ s.t. $F(L, K) = \bar{Y}$

Conditional demand: $\bar{K}(W, R, \bar{Y})$ and $\bar{L}(W, R, \bar{Y})$

$$\frac{F_L(\bar{L}, \bar{K})}{F_K(\bar{L}, \bar{K})} = \frac{W}{R} \quad \text{and} \quad F(\bar{L}, \bar{K}) = \bar{Y}$$

Static model

Two factor inputs: conditional demand elasticities

- Own-price elasticities: $\eta_W^L = \frac{\partial \ln \bar{L}}{\partial \ln W} < 0$ and $\eta_R^K = \frac{\partial \ln \bar{K}}{\partial \ln R} < 0$
- Cross-price elasticities: $\eta_R^L = \frac{\partial \ln \bar{L}}{\partial \ln R} > 0$ and $\eta_W^K = \frac{\partial \ln \bar{K}}{\partial \ln W} > 0$
- Elasticity of substitution $\sigma = \frac{\partial \ln\left(\frac{K}{L}\right)}{\partial \ln\left(\frac{W}{R}\right)} > 0$

It is also possible to show that

$$\eta_R^L = \sigma(1 - s) \quad \text{and} \quad \eta_W^L = -\sigma(1 - s)$$

where $s = \frac{WL}{C}$ is labour share in total cost

Static model

Two factor inputs: unconditional factor demand

Second step: $\max_Y PY - C(W, R, Y)$

Solution: $P = C_Y(W, R, Y^*), L^* = \bar{L}(W, R, Y^*), K^* = \bar{K}(W, R, Y^*)$

Total elasticities decomposed into **substitution** and **scale** effects:

$$\varepsilon_W^L = \eta_W^L + \eta_Y^L \varepsilon_W^Y < 0$$

$$\varepsilon_R^L = \eta_R^L + \eta_Y^L \varepsilon_R^Y \lessgtr 0$$

Estimations of static model

Empirical strategy

1. Shephard's lemma: $\bar{L} = \frac{\partial C}{\partial W} \quad \Rightarrow \quad s = \frac{\partial \ln C}{\partial \ln W}$
2. Specify functional form of $\ln C$

Example: translog cost function with n inputs

$$\ln C = a_0 + \sum_{i=1}^{n} a_i \ln W_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \ln W_i \ln W_j + \frac{1}{\theta} \ln Y$$

3. Regress input share s_i on $\frac{\partial \ln C}{\partial \ln W_i}$
4. Use estimated parameters to compute σ_{ij}

Estimations of static model

Main issues

- Endogeneity
- General equilibrium
- Definitions of variables

Estimations of static model

Review by Hamermesh (1996) concludes that $-\eta_W^L \in [0.15, 0.75]$.

If $\eta_W^L = -0.30$ and given that $s \approx 0.7$,

$$\sigma = \frac{-\eta_W^L}{1 - s} \approx 1$$

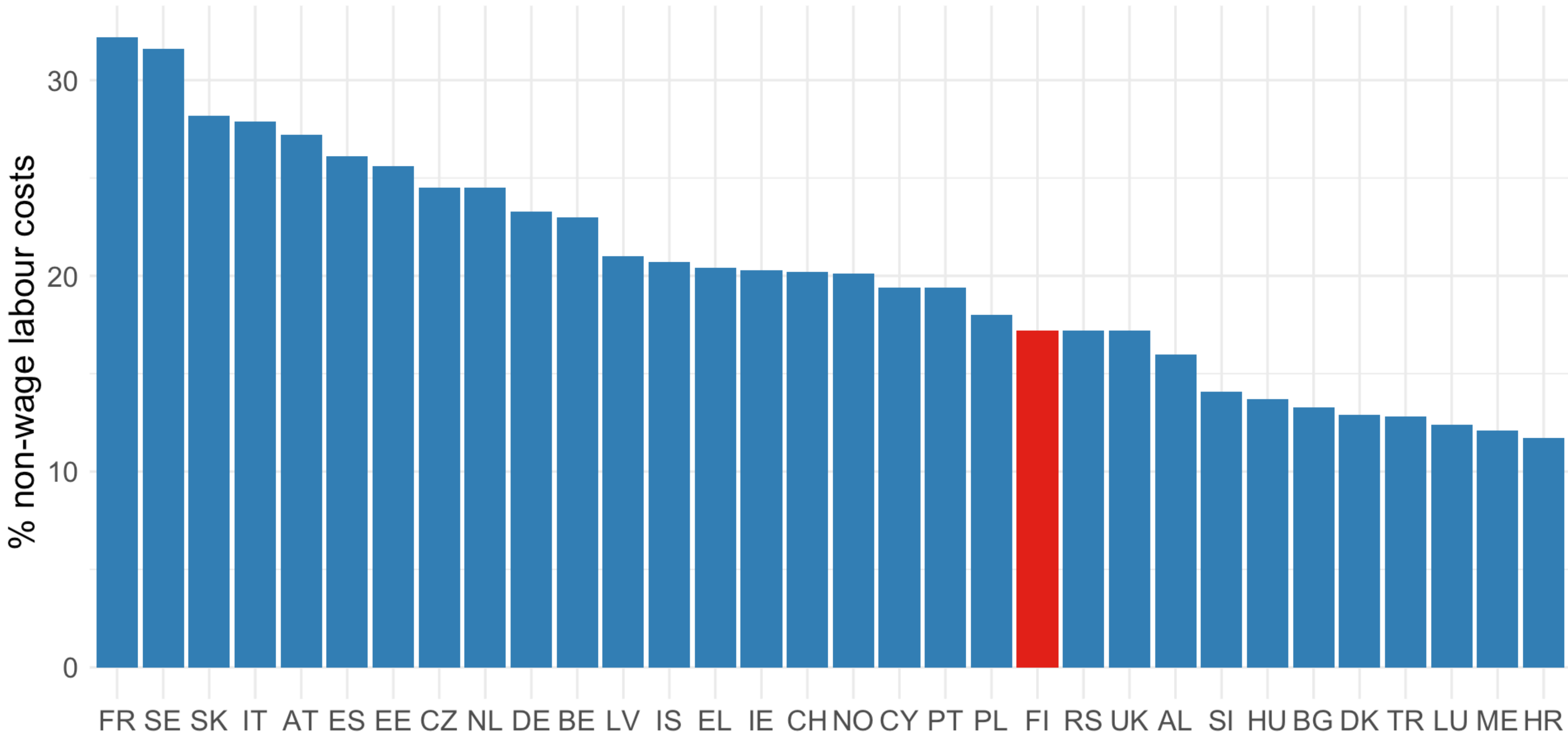
consistent with the Cobb-Douglas production function.

The review also suggests $-\varepsilon_W^L \approx 1 \Rightarrow$ large scale effect.

Dynamic model

Dynamic model

Non-wage labour costs



Source: Eurostat

Dynamic model

Adjustment costs

- Quadratic cost: $C\left(\Delta L_t\right) = b(\Delta L_t - a)^2$
- Asymmetric convex costs: $C\left(\Delta L_t\right) = -1 + e^{a\Delta L_t} - a\Delta L_t + \frac{b}{2}(\Delta L_t)^2$
- Linear cost: $C\left(\Delta L_t\right) = \begin{cases} c_h \Delta L_t & \text{if } \Delta L_t \geq 0 \\ -c_f \Delta L_t & \text{if } \Delta L_t \leq 0 \end{cases}$
- Fixed cost

Dynamic model

Quadratic adjustment cost

- For simplicity, assume single-input: $Y_t = F(L_t)$
- Continuous time: $\Delta L_t = \dot{L}_t = \frac{\mathrm{d}L_t}{\mathrm{d}t}$

$$\Pi_0 = \int_0^\infty \Pi_t dt = \int_0^\infty \left[F(L_t) - W_t L_t - \frac{b}{2}\dot{L}_t^2\right] e^{-rt}\, \mathrm{d}t$$

- Euler equation: $\frac{\partial \Pi_t}{\partial L} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \Pi_t}{\partial \dot{L}_t}\right)$

$$b\ddot{L}_t - rb\dot{L}_t + F'(L_t) - W_t = 0$$

Dynamic model

Quadratic adjustment cost

Optimal path: $\dot{L}_t = \gamma\,[L^* - L_t]$ where γ is decreasing in b.

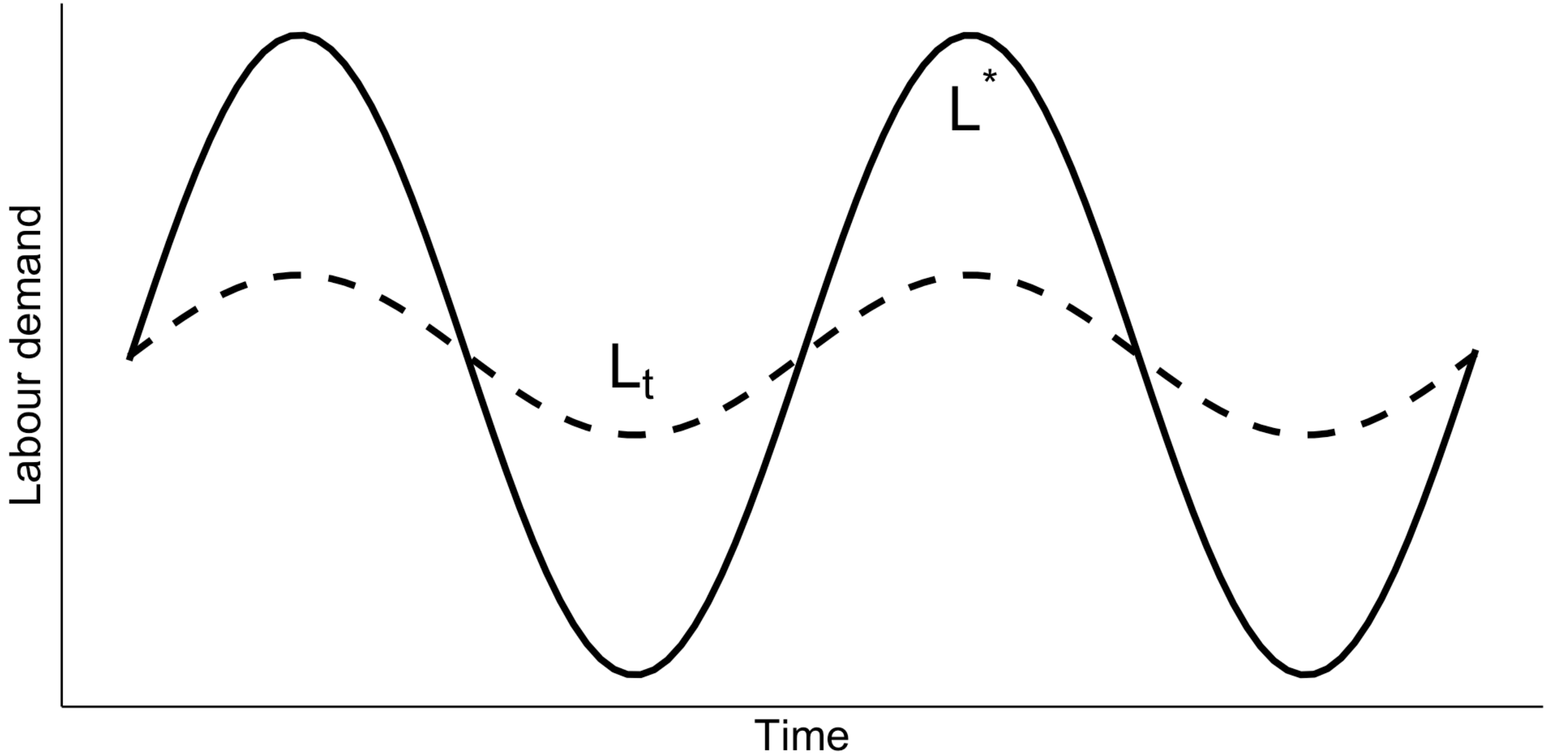


Figure 9.6 Optimal employment over a cycle (Nickell 1986)

Dynamic model

Linear adjustment cost

$$\Pi_0 = \int_0^{\infty} \left[F(L_t) - W_t L_t - C(\dot{L}_t)\right] e^{-rt}\, dt$$

where $C\left(\dot{L}_t\right) = \begin{cases} c_h \dot{L}_t & \text{if } \dot{L}_t \geq 0 \\ -c_f \dot{L}_t & \text{if } \dot{L}_t \leq 0 \end{cases}$

Optimal labour demand path is derived from

$$\begin{cases} F'(L_t) = W_t + rc_h & \text{if } \dot{L}_t \geq 0 \\ F'(L_t) = W_t - rc_f & \text{if } \dot{L}_t < 0 \end{cases}$$

Dynamic model

Linear adjustment cost

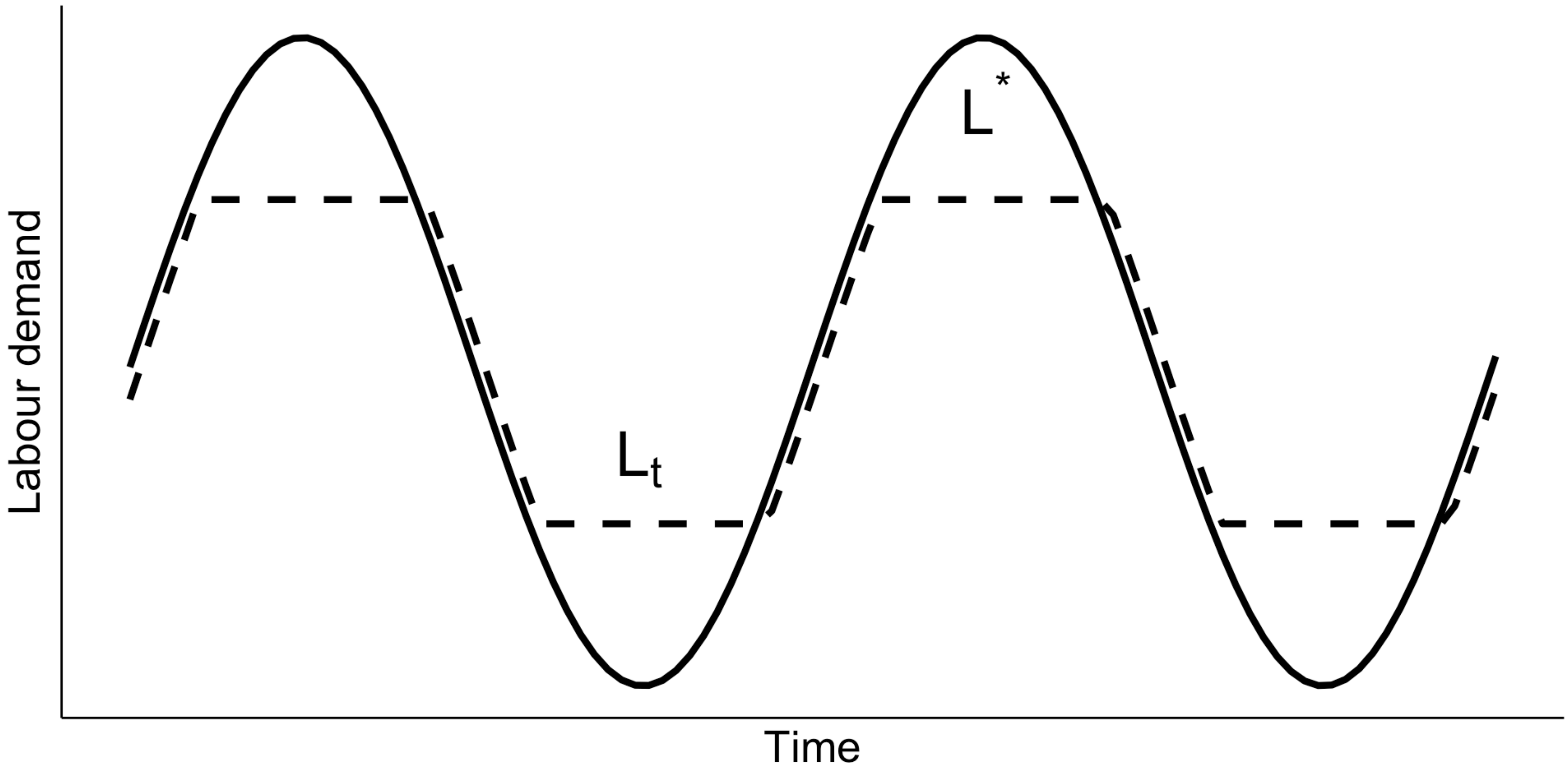


Figure 9.10 Optimal employment over the cycle (Nickell 1986)

Estimations of dynamic model

Empirical strategy for adjustment cost specification

Quadratic adjustment cost

- Assume linear quadratic production function
- Estimate $L_{it} = \lambda L_{i,t-1} + X_{it}\beta + \mu_i + \varepsilon_{it}$

  → Need to account for $\mathbf{Corr}\left(L_{i,t-1}, \mu_i + \varepsilon_{it}\right)$

Estimations of dynamic model

Some key results

- Adjustments happen fast (1-2 quarters) (Hamermesh 1996, chap. 7)
- Dynamic substitutes: utilization of capital increases with $L_t - L^*$
- Hours of work are adjusted faster than number of workers

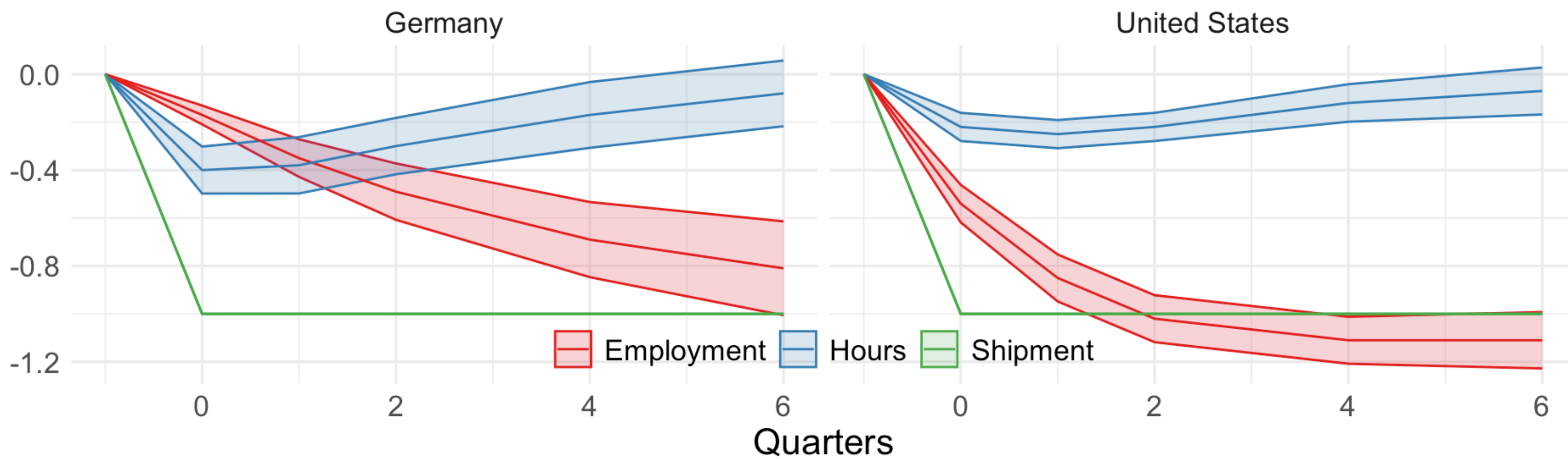


Figure 1 from Houseman and Abraham (1993) (adjustment to demand shocks)

Minimum wages and employment

Minimum wage and employment

What do the models we have considered so far predict?

- lower labour demand (both compensated and uncompensated)
- (maybe) higher labour supply

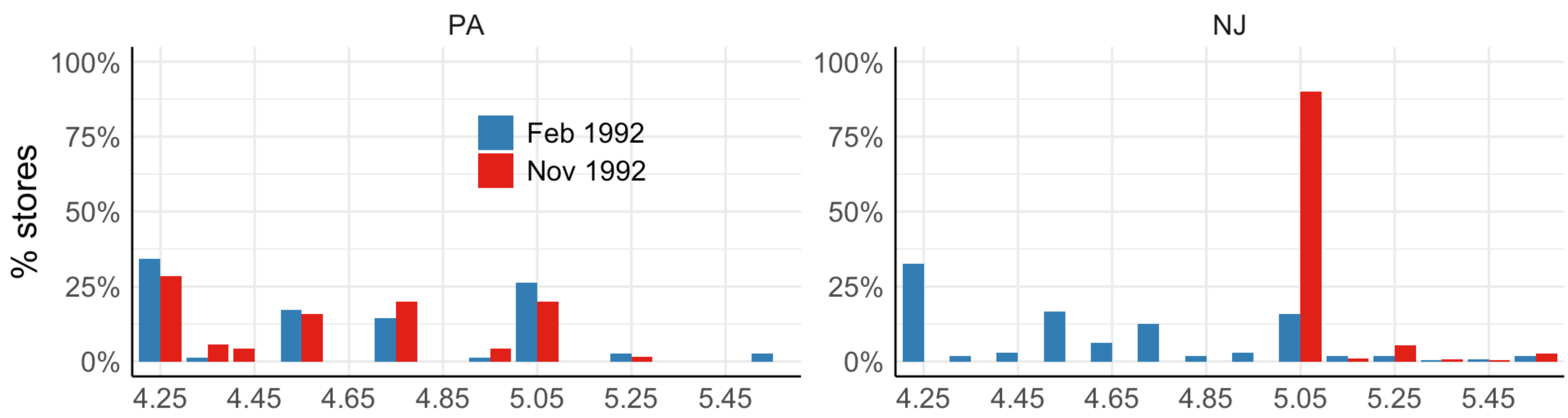
Not always supported by empirical evidence!

Minimum wage and employment

*Card and Krueger (**1994**)*

On April 1, 1992 minimum wage in New Jersey ↑ from \$4.25 to \$5.05.

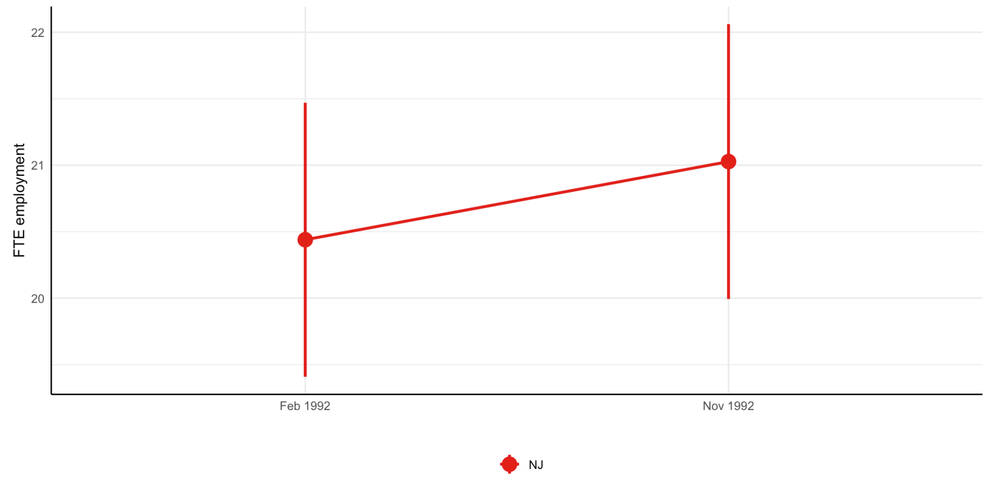
It stayed at \$4.25 in Pennsylvania.



Minimum wage and employment

*Card and Krueger (**1994**): Difference-in-differences*

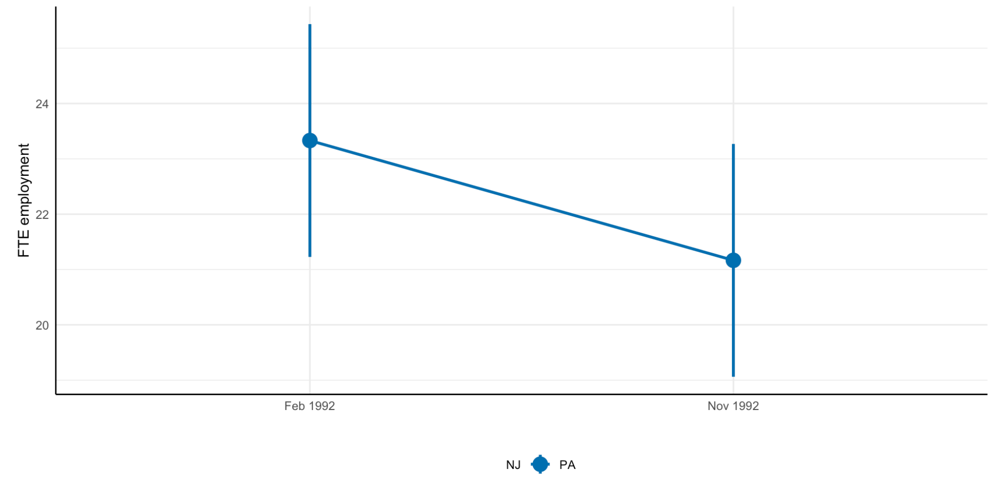
1. Compare before and after in NJ:
   $E_{t1}^{NJ} - E_{t0}^{NJ}$ = 0.59 (se = 0.73)



Minimum wage and employment

*Card and Krueger (**1994**): Difference-in-differences*

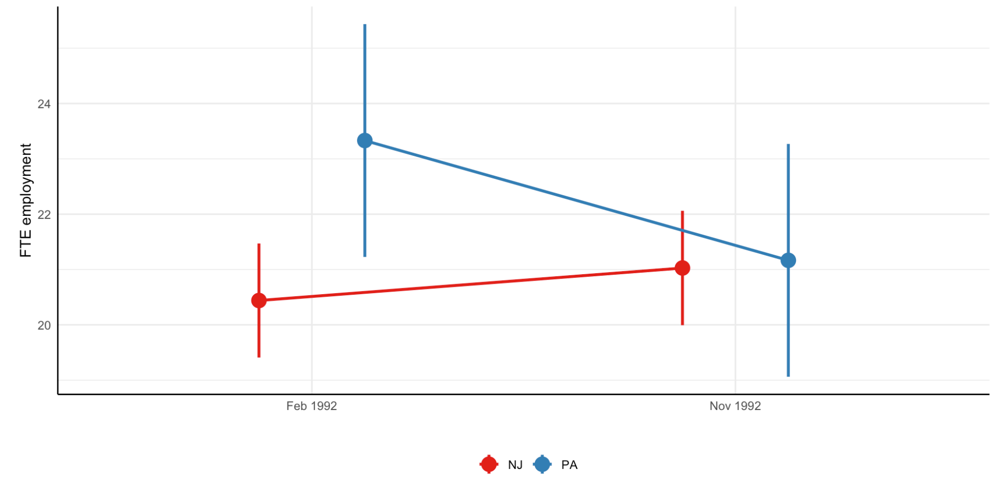
1. Compare before and after in NJ:
   $E_{t1}^{NJ} - E_{t0}^{NJ}$ = 0.59 (se = 0.73)
2. Compare before and after in PA:
   $E_{t}^{NJ} - E_{t}^{PA}$ = -2.17 (se = 1.65)



Minimum wage and employment

*Card and Krueger (**1994**): Difference-in-differences*

1. Compare before and after in NJ:
   $E_{t1}^{NJ} - E_{t0}^{NJ}$ = 0.59 (se = 0.73)
2. Compare before and after in PA:
   $E_{t}^{NJ} - E_{t}^{PA}$ = -2.17 (se = 1.65)
3. Diff-in-diff:
   $\left(E_{t1}^{NJ} - E_{t0}^{NJ}\right) - \left(E_{t1}^{PA} - E_{t0}^{PA}\right)$ =
   2.75 (se = 1.69)



Minimum wage and employment

*Jardim et al. (**2022**)*

Seattle ↑ min wage from $9.47 up to

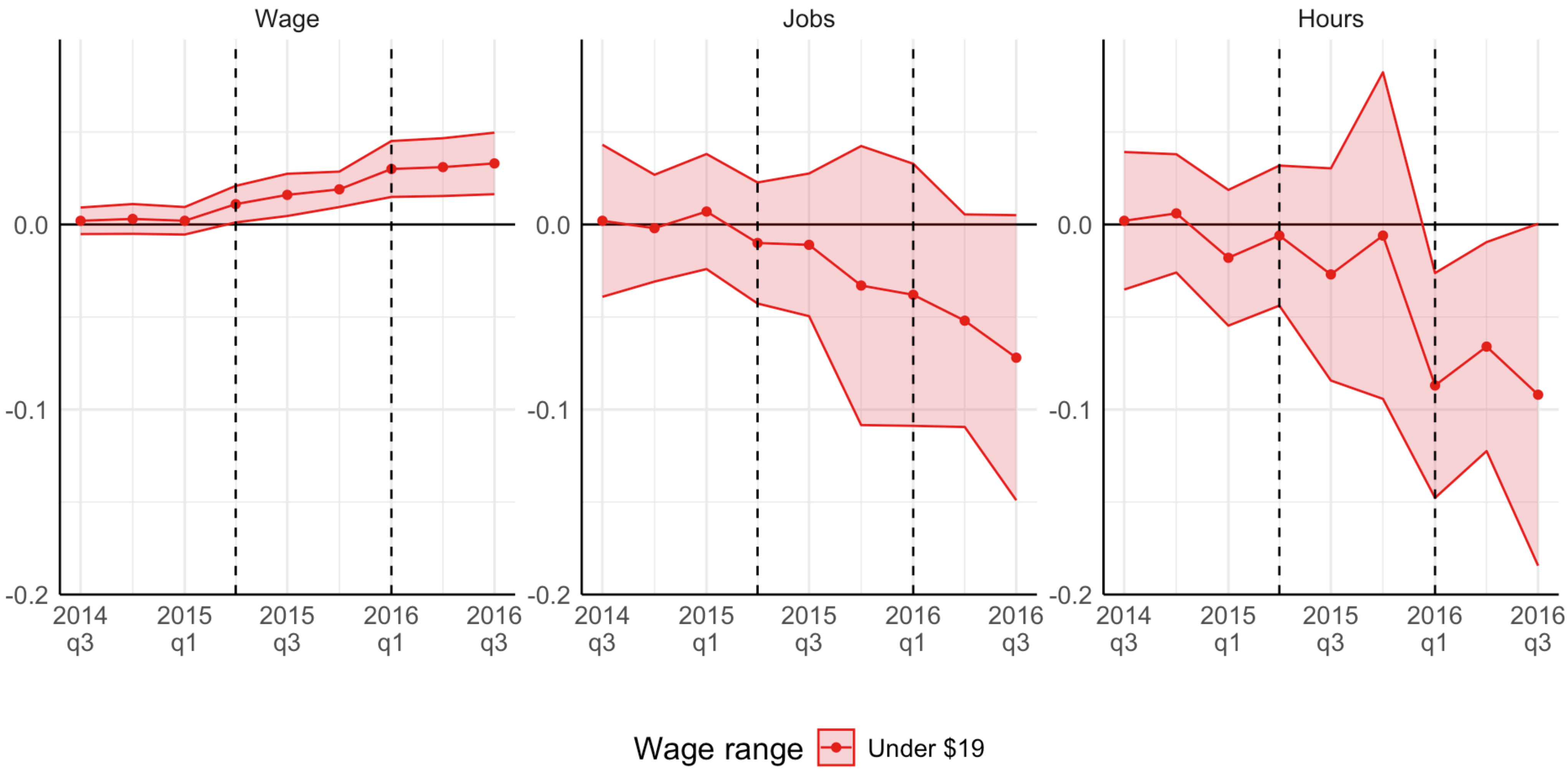
- $11 in April 2015
- $13 in January 2016

Causal design:

- **synthetic control**: weighted average of other counties match pre-Seattle
- **nearest neighbour matching**: find "closest" worker outside of Seattle matching treated worker in Seattle

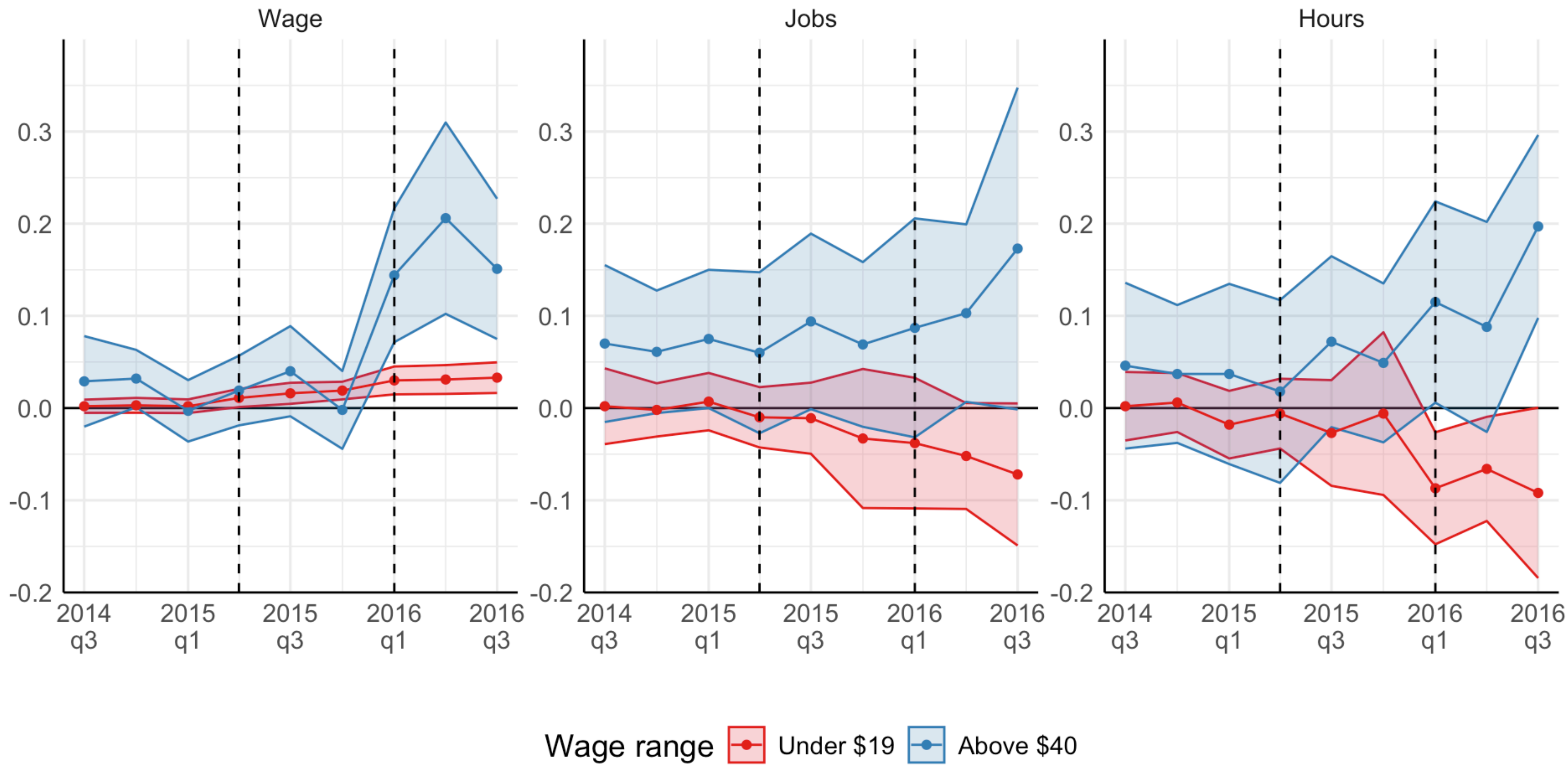
Minimum wage and employment

*Jardim et al. (**2022**): synthetic control*



Minimum wage and employment

*Jardim et al. (**2022**): synthetic control*



Minimum wage and employment

*Jardim et al. (**2022**)*

- Negative effect on hours worked stronger than on employment
- Experienced workers are better off

However,

- Potentially cascading effect
- Excluded large low-wage employers (like McDonald's)

*Reich, Allegretto, and Goddy (**2017**)*

same policy + synthetic control = no change in employment

Minimum wage and other margins

Review in Clemens (2021)

- Price pass-through (Leung 2021; Renkin, Montialoux, and Siegenthaler 2022)
- Non-wage labour cost (Clemens, Kahn, and Meer 2018)
- Flexibility (theoretical Clemens and Strain 2020)
- Effort (Ku 2022; Coviello, Deserranno, and Persico 2022)
- Firm profit (Draca, Machin, and Van Reenen 2011; Bell and Machin 2018)
- Firm exit (Luca and Luca 2019; Dustmann et al. 2022)

Summary

- Basic static and dynamic models of labour demand
- Application to minimum wage policy
  - → Ongoing research (little consensus)
  - → Clear that basic models are insufficient
  - → Non-wage margins important and can interact with labour supply

Next lecture: Job Search on 03 Sep

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